MTL 106 (Introduction to Probability and Stochastic Processes)

II Semester 2016-17

Tutorial Sheet 1

Sigma Field, Independence and General Probability

1. Let Ω = {a,b,c,d}. Find three different σ-fields {Fn} for n = 0, 1, 2 such that F0 ⊂F1 ⊂F2
2. Let Ω = {0,1,2, …}. Let F be the collection of subsets of Ω that are either finite or whose complement is finite. Is F a σ-field? Justify your answer.
3. Consider Ω = {(x, y) : 0 ≤ x ≤ 1, 0 ≤ y ≤ 1}. Let F be the largest σ-field over Ω. Define P(R) = area of R = (b - a)(d - c) where R is the rectangular region that is a subset of Ω of the form R = {(u, v) : a ≤ u < b, c ≤ v < d}. Let T be the triangular region T = {(x, y): x ≥ 0, y ≥ 0, x + y < 1}. Show that T is an event and find P(T), using axiomatic definition of probability.
4. State True or False:
   1. Let Ω = {a,b,c}. If F1 = {Φ, {a}, {b,c}, Ω} and F2 = { Φ, {a,b}, {c}, Ω} are two sigma fields on Ω then F1 ∪ F2 and F1 ⋂ F2 are σ fields on Ω
   2. A box contains a double-headed coin, a double-tailed coin and an unbiased coin. A coin is picked up at random and flipped. It shows a head. The conditional probability that it is a double headed coin is 0.5
   3. Let A and B be two events with P(A) = ½ and P(Bc) = ¼. Then A and B can be mutually exclusive events.
5. Consider the flights starting from Delhi to Bombay. In these flights, 90% leave on time and arrive on time, 6% leave on time and arrive late, 1% leave late and arrive on time and 3% leave late and arrive late. What is the probability that, given a flight is late, it will arrive on time.
6. Let A and B are two independent events. Prove or disprove that A and Bc, Ac and BC are independent events.
7. In a meeting at the UNO 40 members from under-developed countries and 4 from developed ones sit in a row. What is the probability no two adjacent members are representatives of developed countries.
8. An electronic assembly consists of two subsystems, say A and B. From previous testing procedures, the following probabilities are assumed to be known: P(A fails) = 0.20, P(A and B both fail) = 0.15, P(B fails alone) = 0.15. Evaluate the following probabilities: (a) P(A fails / B has failed) (b) P(A fails alone / A or B fail)
9. Along a line segment ab two points l and m are randomly marked. Find the probability that l is closer to a than m, (al < am)
10. An urn contains b black balls and r red balls. One of the ball is drawn at random, but when it is put back in the urn c additional balls of the same colour are put in with it. Now suppose that we draw another ball. Find the probability that the first ball drawn was black given that second ball drawn was red?
11. The base and altitude of a right triangle are obtained by picking points randomly from [0, a] and [0, b] respectively. Find the probability that the area of the triangle so formed will be less than ab/4 ?